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AN ANALYSIS OF THE POSITION LOCATION AND REPORTING SYSTEM'S PERFORMANCE CHARACTERISTICS

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THESIS

AN ANALYSIS OF THE POSITION LOCATION AND
REPORTING SYSTEM'S PERFORMANCE
CHARACTERISTICS

by

Walter Woodrow Sevon Jr.

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Thesis Advisor: D. R. Barr

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and Stepwise Linear Regression on test data from the two systems.

An Analysis of the Position Location
and
Reporting System's Performance Characteristics

by

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The variables which significantly affect accuracy in two position location and reporting systems are examined in this thesis. Some physical and mathematical characteristics of the two systems are described. A model is developed to evaluate system accuracy. The significant variables are identified using the techniques of Analysis of Variance and Stepwise Linear Regression on test data from the two systems.

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I. INTRODUCTION

A. GENERAL

The purpose of this research was to identify the variables that significantly affect accuracy in the Position Location and Reporting System (PLRS) and to determine the nature of their effects. A model was developed to assist in identifying the variables and to predict system accuracy as a function of various conditions.

Section I of this thesis explains the fundamental principles of both the Hughes Aircraft Company PLRS and the General Dynamics PLRS. The development of the form of the model and the procedure used for testing the model is found in Section II. The results of the testing and the analysis of the data are presented in Section III. Conclusions and recommendations are presented in Section IV.

Differences in the design and operation of the two PLRS's are discussed. Actual test data is used to identify the variables that significantly affect system accuracy. From these test data differences in performance between the two PLRS types are identified. Recommendations for future testing are provided, which it is hoped will be of use in the next stage of the PLRS's development and testing.

B. BACKGROUND

The PLRS is designed to provide continuous and accurate locations of both aircraft and ground units and to allow a limited amount of communications between these units. The PLRS is also designed to provide the bearing and the range from any PLRS unit to any pre-designated point within the PLRS operational area depicted in Figure 1. In Figure 1, Area A is considered the "primary ground area of operation," in which the ground units are located. Area B primarily concerns aircraft units which are either entering or leaving Area A [1].

The PLRS is a system under joint development by the United States Marine Corps and the United States Army. The Engineering Development Model (EDM) Tests were completed at the Marine Corps Tactical Systems Support Activity (MCTSSA), Camp Pendleton, California in December, 1975. Two candidate EDM systems, one developed by Hughes Aircraft Company (HAC) and the other by General Dynamics Corporation (GDC) were tested to determine if they met contract specifications and to determine the "best" system for possible advancement to the next stage of development. Readers familiar with PLRS may bypass Section I.C. thru Section I.E. of this thesis and proceed directly to Section II.

C. SYSTEM DESCRIPTION

1. General Concept

The PLRS is a network composed of a main computing facility and as many as 370 smaller user units. Communication is maintained between these components by radio messages.

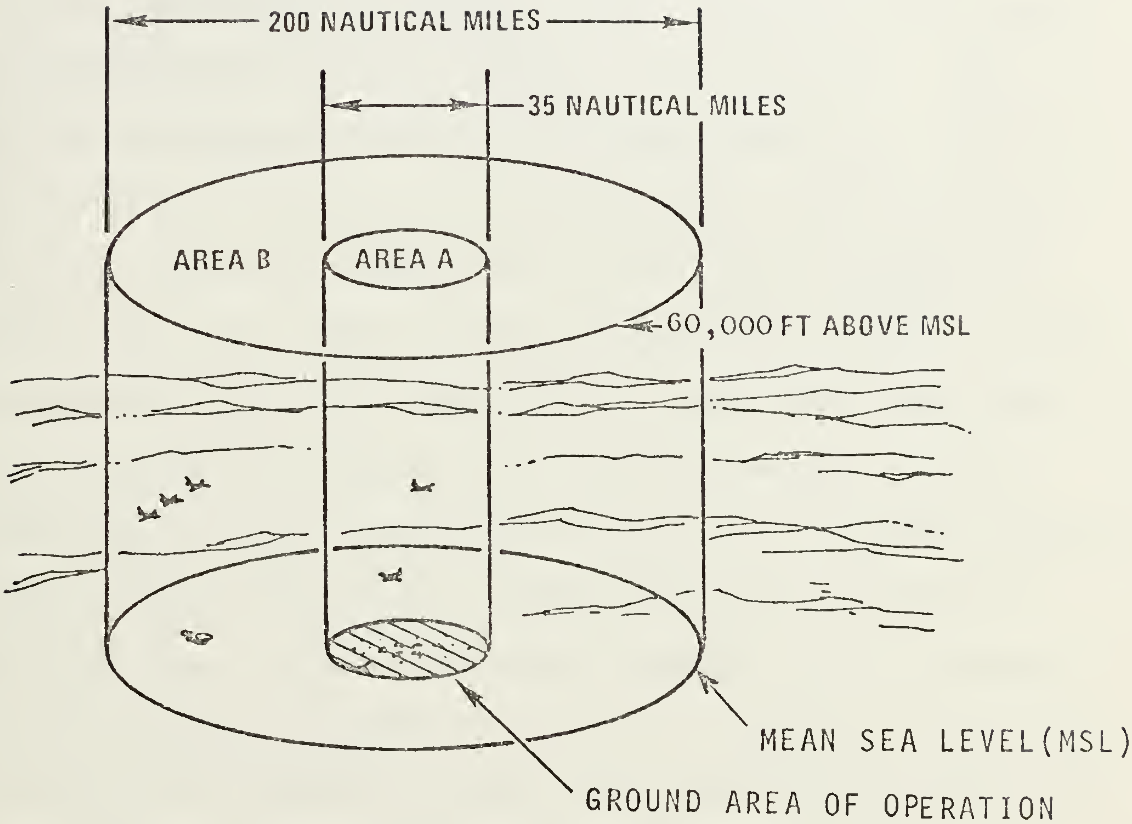

FIGURE 1. The PLRS Operational Area [1]

Such messages are passed only between line-of-sight (LOS) members of the network. One unit is considered LOS to another unit if there is no physical object between the units which interrupts communication between them. Messages may contain ranging information between members in the network. This ranging information is passed to the main computing facility which uses it to estimate the locations of network members. Units LOS to the main computing facility relay messages to and from user units which are non-line-of-sight (NLOS) to the main computing facility. Relay chains are formed when more than one user unit is employed to relay messages between a user unit and the main computing facility. A user unit LOS to the main computing facility is considered to be located at relay level zero. A user unit that requires only one other user unit to relay messages to the main computing facility is considered to be located at relay level one. User units are located at relay levels two and three if there are two and three other user units respectively between them and the main computing facility.

2. System Components

a. Master Unit (MU)

The MU houses the computer facilities as well as the communication control electronics. It controls transmissions between all the PLRS user units and continuously monitors and displays user unit locations and user unit movements as they occur. It automatically establishes relay

chains to maintain network communications with all NLOS elements of the PLRS network [1].

b. User Unit (UU)

The UU is a multifunctional, lightweight item of field equipment, operable either with batteries or vehicle power. It is designed to automatically perform certain functions within the PLRS network necessary to produce range measurements between units, pass communications between units, perform relay functions, and enter or display position and other information to the user unit operator for his use in positioning and navigation. Each unit is designed to have the capability to transmit and receive up to 100 prespecified digital messages from the MU. Each UU has an altimeter which measures and reports its altitude to the MU. UU's are assembled in four configurations.

1. Manpacked (MPU),
2. Surface Vehicle Mounted (SVU),
3. Aircraft Mounted (AU), and
4. Fixed Reporter (FRU).

A FRU is a UU with known position.

3. Position Determination

Both the HAC PLRS and the GDC PLRS use the methods of trilateration and multilateration (the repeated application of trilateration) to determine a unit's position. Trilateration is a technique used to resolve a triangle through knowledge of the length of its sides, rather than of its included angles as is done in triangulation. The use of this technique in

the PLRS requires that the system have the capability of "accurately" measuring the distance between any two LOS units. The system must also be able to relate the calculated positional information to a suitable external reference or coordinate system. The distance measurement concepts for the two contractors are not the same and are discussed below.

a. Distance Measuring Concepts

(1) Hughes Aircraft Company (HAC)

Distance measurements in the HAC PLRS are made using a time ordered system. In this time ordered system, only one unit is allowed to transmit during a single time increment. To ensure that this happens, the units of the system share a common time base, and a clock located in each UU is synchronized to a master clock in the MU. The distance calculations are made by the MU which directs each UU to transmit at a precisely designated instant during one of the time increments. All other units attempt reception during such transmission. Each unit receiving the transmission reports the measured time of reception to the MU, and the MU then estimates the time it took for the signal to reach each receiver. From these times and an estimate of the speed of the radio signal through the atmosphere, the MU can perform calculations for estimating the distance separating the unit transmitter and each of the receiving units [2].

(2) General Dynamics Corporation (GDC)

Each UU in the GDC PLRS can function as a Relay, a Reporter, or a Responder. If commanded by the MU to function as a Relay, the unit merely relays a message to another UU. When a Reporter Unit function is specified, the unit measures the time it takes to send a signal to a designated unit and for that unit to return the signal. From this time and an estimate of the speed of the signal through the atmosphere, the MU can estimate the distance separating the Reporter Unit and the designated unit. When performing as a Responder Unit, the unit either accepts or displays data received by the MU, formats and transmits data requested by the MU, or transmits a range pulse to a reporter. The method of using units to function as Reporters, Relays or Responders is called the Interrogate -Transpond Technique [1].

b. Position Location Smoothing

When more than three range measurements are made, the redundancy in data is used to provide a multilateration solution. This solution provides many estimates of a unit's position which must be combined to place the unit at a single location. Smoothing techniques are used to arrive at a single estimate. Smoothing may be said to be the application of averaging techniques to the task of obtaining reliable estimates of position. Smoothing techniques are commonly referred to as error reduction techniques or variance reduction techniques. Their purpose is to reduce the variance of the calculated

position of a unit around the "true" location of the unit. The HAC PLRS and the GDC PLRS use different smoothing techniques. The HAC PLRS uses a form of the least squares smoothing technique. The GDC PLRS uses the Kalman filtering smoothing technique. A discussion of the differences in these techniques is found in Tysver, Demetry and Haworth [3].

D. STATEMENT OF THE POSITION LOCATION PROBLEM

The distances between UU's calculated by the MU are estimates which necessarily include some error. These estimates are used in the trilateration/multilateration technique to position locate a unit. The quality of these estimates is directly affected by such factors as a unit's speed, altitude and relay level. Other factors discussed also affect the quality of these estimates. In addition, the HAC PLRS and the GDC PLRS use different methods to estimate these distances and use different smoothing techniques to arrive at the position determination for a unit. In this thesis the author attempts to determine the factors, by contractor, that significantly affect system accuracy and the nature of their effects.

E. PLRS ENGINEERING DEVELOPMENT MODEL TEST PLAN

One of the three main purposes of the PLRS EDM test plan was to determine if the EDM's met the contract specifications on accuracy [4]. The method used was to compare the accuracy of the PLRS EDM systems under certain operating conditions. The testing consisted of a number of subtests. Both PLRS EDM

systems were comprised of a MU and seventeen UU's. UU's were classified as MPU's, FRU's, AU's or SVU's as directed by each subtest. Not all types of units were used in every subtest. The MPU's and FRU's were placed on surveyed positions during the subtests to establish their "true" positions. AU's and SVU's were tracked by a laser tracker to establish their "true" position. The PLRS EDM computer software kept time tagged magnetic tape records of each estimated UU position. A program tape was run against the magnetic tape records of each subtest which measured the differences between a UU's estimated position and its "true" position. These residuals were used to calculate system accuracy. The "true" location of each UU was established by one of the control items listed below [4].

1. Test Site Survey

Ground UU locations were surveyed and monumented by the Geodesy Survey Team from the Pacific Missile Range, Pt. Mugu. The positions were positioned as first order survey points.

2. Laser Trackers

Dynamic short range air and ground instrumentation was provided by the Mobile Automatic Laser Tracking System (MALTS). The MALTS provided time synchronized position locations for dynamic units accurate to two meters at 9000 meters to .6 meters at 2000 meters.

3. Data Reduction

The PLRS estimate of a UU's position for dynamic units was compared with the MALTS estimate. For static UU's, the PLRS estimate was compared with "true" estimate input on the PLRS program tape written by PLRS test team personnel. These residual differences were used to calculate system accuracy.

II. EXPERIMENTAL PROCEDURE

A. GENERAL

The purpose of this thesis was to identify the variables that significantly affect accuracy in the PLRS and to determine the nature of their effects. In this section, a Measure of Effectiveness for accuracy is proposed and defined. The sources of error which affect accuracy in the PLRS are examined. A model is then developed which relates accuracy to the variables expected to affect accuracy.

B. MEASURE OF EFFECTIVENESS (MOE)

The author chose the Circular Error Probable (CEP) of a units location as the MOE to reflect system accuracy. CEP is the same MOE that was used in the PLRS EDM test plan. CEP is defined as the radius of a circle which on the average contains fifty percent of the position location solutions for a particular user unit. CEP is defined only for two dimensions, and is a projection on the ground plane for an airborne unit. The CEP is divided into two categories, Circular Error Probable Accuracy (CEP_A) and Circular Error Probable Precision (CEP_P). CEP_A is based on a circle centered at the "true" location of the user unit. CEP_P is based on a circle centered at the "mean" location of the user unit. Here, the "true" position of the user unit was assumed to be the surveyed position coordinates for static units and the position coordinates

provided by the MALTS for dynamic units. This assumption is very critical to this analysis. If the position coordinates provided either by survey or by the MALTS are not accurate, the CEP will be calculated incorrectly, bias will be introduced in the CEP, and incorrect conclusions will possibly be drawn from the analysis. During the PLRS EDM testing, CEP_P was estimated using a method described by Swinburne [5]. CEP_A was estimated using a method described by Valstar [6].

C. SOURCES OF POSITION LOCATION ERROR

The sources of error in the PLRS which the author believes may be of primary importance are discussed below. These errors either affect range measurement accuracy or the accuracy of the position location solution.

1. Resolution of Equipment

The user units are designed to time the transmission of radio messages from one user unit to another. This time cannot be measured without error, even with high resolution equipment. This error contributes to the range measurement error. This range measurement can be considered a random variable. Its expected value is assumed to be the true range measurement. Its variance, σ^2, is assumed to be constant over test conditions [4]. This variance has been estimated at four meters for the GDC PLRS [1].

2. Geometry

"Geometry" is a general term used to describe the overall position of UU's relative to other UU's and the MU.

The aspect angle, α , is a specific measure of geometry. It is used to describe the angular position of a UU relative to other LOS UU's. It is defined as the maximum angle at a UU subtending the arc of other LOS UU's. An example is illustrated in Figure 2. A 360^{o} aspect angle occurs either when there are LOS UU's surrounding the UU such that no angle exists greater than 90^{o} between adjacent UU's, or when there is at least one airborne unit in the network LOS to the UU. Burt, et al. [7] has shown that for three UU's the geometry contribution to CEP is at a minimum when the aspect angle is 90^{o}. Figure 3 depicts a curve of CEP magnification as it relates to aspect angle. When more than three UU's are involved and the aspect angle increases beyond 180^{o}, the exact relationship between the aspect angle and CEP magnification is not clear. However, one would expect that if both the aspect angle increase and the number of UU's LOS increase, the CEP would decrease.

3. Number of Units LOS

As the number of units LOS to a particular UU increases, the number of ranges used in the multilateration position location solution also increases. Lee [8] states that typical errors in accuracy should be proportional to $1/\sqrt{n}$, with n being the number of units LOS. This implies, for example, that to halve CEP, it would be necessary to increase the number of units LOS by a factor of four.

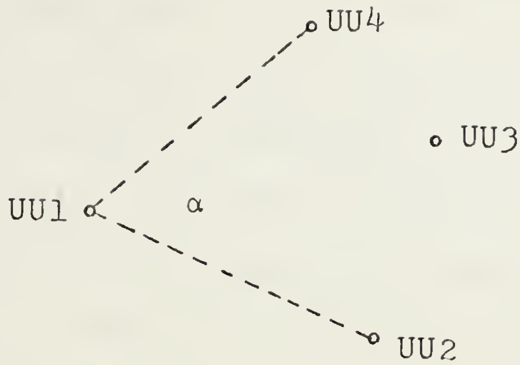

FIGURE 2. Aspect Angle. UU's 2, 3 and 4 are all LOS to UU1. The aspect angle, α , subtends the arc from UU2 to UU4.

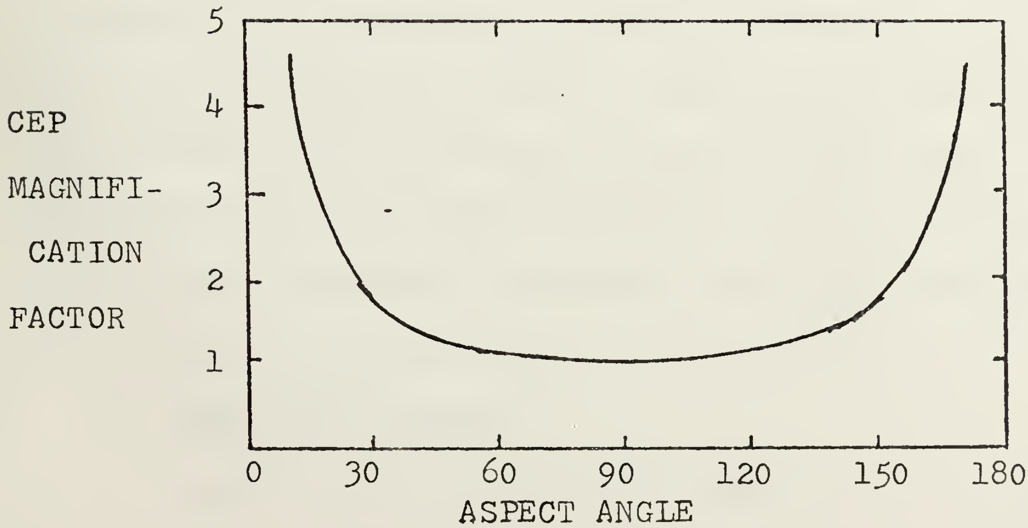

FIGURE 3. CEP magnification vs. aspect angle [7]

4. Relay Level

The relay level of a UU was defined in Section I. UU's located at relay level zero are position located, in most cases, by range measurement estimates provided by the FRU's and the MU. The CEP of these units can then be calculated using only the range measurement variance and the aspect angle. These UU's located at relay level zero provide range measurement estimates for the position location solution of those UU's located at relay level one. To compute the CEP for the units located at relay level one, the CEP of the UU's providing range measurement estimates must be considered in addition to the range measurement variance and aspect angle. The CEP^2 is a measure of uncertainty in a sense like the range measurement variance. An adjusted range measurement variance can be formed by summing the range measurement variance and the underlying CEP^2 of the UU providing the range measurement estimate. With this adjusted range measurement variance and the aspect angle, the CEP for units at relay level one can be estimated. This same procedure can be used for UU's located at relay levels two and three. Hence, UU's located at higher relay levels will be position located by UU's with larger adjusted range measurement variances and thus will have larger CEP's. This procedure of summing the range measurement variances is recommended by Burt, et al. [7].

5. Speed and Altitude

Lee [8] states that for PLRS which are primarily ground-based, errors are primarily altitude errors. The

combined effects of a UU's speed and altitude on that UU's accuracy is not clear to the author. Subtests in the PLRS EDM testing were designed to test accuracy with airborne units at different levels of speed and accuracy. Data from these subtests is available to test the effects of both a UU's speed and altitude on accuracy.

D. MODEL DEVELOPMENT

1. General

A model form for system accuracy is proposed in this section. The model form is based as much as possible on the mathematical and physical characteristics of a PLRS. Unknown parameters are included in the model which are estimable from the test data. The purpose of developing a model is two-fold. First, to use the model to assist in identifying the effects of certain variables on accuracy. Secondly, for use in predicting UU accuracy as a function of the significant variables.

2. Mathematical Considerations

CEP was defined as the radius of a circle which on the average contains fifty percent of the position location solutions for a particular user unit. The position location solutions could be considered as samples of a random vector. Let us call this random vector X. The distribution of X is dependent upon the aspect angle and the range measurements. If the aspect angle is 90° and the range measurement variances are equal then X is described by the Circular Normal Distribution and the CEP is at a minimum [7]. In the general

case X is described by the General Bivariate Gaussian Distribution [7]. When the aspect angle is not 90°, CEP increases. The parameters of the distribution of X are functions of the range measurements which "fix" a position, and the correlation between the range measurements. For the PLRS under consideration, the correlation between the range measurements was assumed to be zero [4]. This assumption does not imply that the variance covariance matrix for X is diagonal. Geometrical relationships will introduce non-zero covariance terms in the Bivariate Normal Distribution. The distribution function is characterized by concentric ellipses of equal probability density [6]. The CEP calculations are not as straightforward for the elliptical normal case as in the case of the circular normal case, i.e. circular contour figures. Procedures for calculating CEP for the bivariate distribution are found in Valstar [6]. Figure 4 shows an intersection of two range measurements with differing range measurement variances.

3. Initial Model

The model initially proposed to describe unit accuracy as measured by the CEP was of the following form:

$$CEP_i^2 = \frac{k(r_i + 1)}{n_i \sin \alpha^*}$$

where

$CEP_i^2 = CEP^2$ of the i^{th} UU

k = range measurement variance

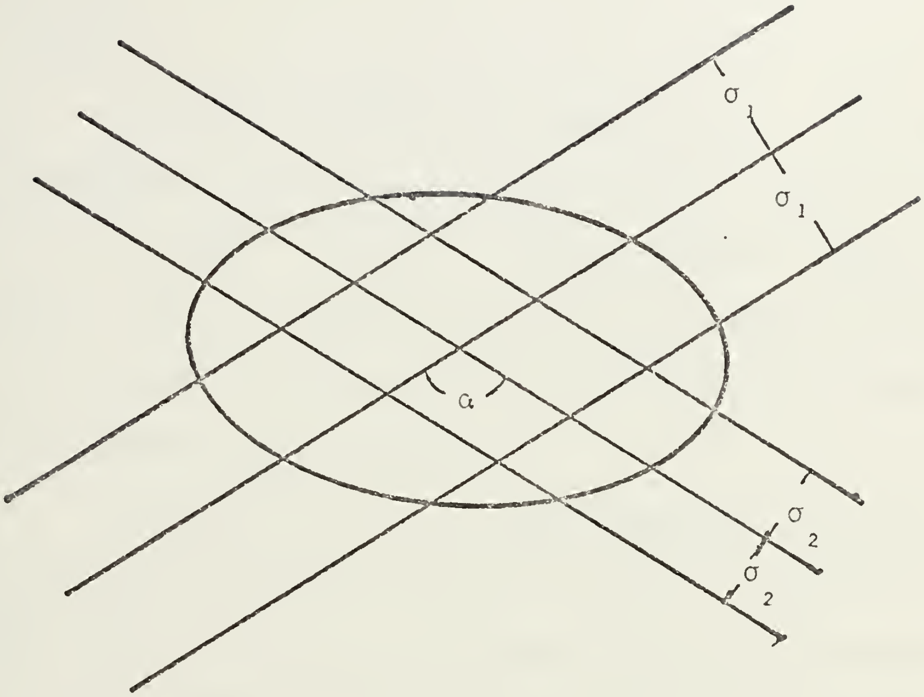

FIGURE 4. Expanded view of the intersection of two range measurements with a sample error ellipse.

r_i = relay level of i^{th} UU

$$\alpha^* = \frac{\alpha}{4}$$

n_i = number of UU's LOS to the i^{th} UU

This model form has intuitive appeal for the following reasons:

a. The area of the parallelogram shown in Figure 4 is $\dfrac{\sigma_1 \sigma_2}{\sin \alpha}$. This area is approximately proportional to CEP^2.

This suggests the basic form of the model to be:

$$CEP_i^2 = \frac{\sigma_1 \sigma_2}{\sin \alpha} .$$

b. As α goes to 90^o, sin α goes to one. This implies CEP^2 is minimum when the aspect angle is 90^o. When α is allowed to go to 360^o, the sin α changes sign. Using α^* is consistent with the expectation that the CEP decreases while α increases as discussed in Section II.C.2. The $\sin \alpha^*$ then varies between zero and one.

c. In Section II.c.4, it was suggested that the adjusted range measurement variance be used to calculate a unit's CEP. The term $k(r_i + 1)$ takes the range measurement variance, k, and sums it over relay levels. This provides an adjusted range measurement variance which is used to estimate a unit's CEP.

d. The number of units LOS, n, is believed to impact upon the CEP as $1/\sqrt{n}$, as was mentioned in Section II.C.3. This implies that as the number n of the units LOS increase, CEP^2 should decrease as $1/n$.

E. THE PLRS EDM TEST DATA

The data available from the PLRS EDM testing was divided into the following categories: contractor, CEP_A, CEP_P, relay level, number of units LOS, aspect angle, number of FRU's, speed and altitude. Each data point was itself an average of between 15 and 500 separate smoothed position determinations

of a UU. The data was separated into four subsets for examination. The data was separated by contractor, and the number of data points for each set is found in Table I. The data sets were subdivided as follows:

1. Data Set 1

   All the data was included in this subset.

2. Data Set 2

   This includes only the data on airborne units.

3. Data Set 3

   This is a basic set of data where the number of FRU's is held fixed at two. Data on airborne units is not included.

4. Data Set 4

   The data in this data set came exclusively from a sub-test which tested the effect of the aspect angle on the PLRS accuracy. All UU's were LOS to the Master Unit and the aspect angle varied between 5° and 90°.

Data sets 2, 3 and 4 are all subsets of Data Set 1. The sample means of the CEP_A and the CEP_P with their estimated standard deviations are presented in Table II. Examination of this data shows a proportional relationship between the sample mean and the sample standard deviation. The larger sample means of the CEP have the larger sample standard deviations and the smaller sample means have the smaller sample standard deviations.

Data Set	Number of Data Points	
	HAC PLRS	GDC PLRS
1	290	368
2	43	74
3	98	119
4	72	72

TABLE I. Data Set subdivision by contractor.

Data Set		HAC PLRS		GDC PLRS	
		$\overline{CEP}$	S	$\overline{CEP}$	S
1	CEP_A	14.8	13.7	22.3	22.8
	CEP_P	8.9	9.1	12.7	20.6
2	CEP_A	22.64	10.88	22.33	12.56
	CEP_P	18.47	11.04	14.85	11.48
3	CEP_A	13.8	9.53	22.3	16.5
	CEP_P	8.78	7.3	7.43	6.98
4	CEP_A	14.63	11.48	34.8	40.1
	CEP_P	5.04	3.80	27.74	39.32

TABLE II. The CEP sample mean and sample standard deviation, S, for all data sets.

F. DATA ANALYSIS PROCEDURE

1. General

Two techniques for analyzing the data are presented in this section. The techniques presented are Analysis of Variance (AOV) and Stepwise Linear Regression. The data lends itself to analysis by these techniques. The data available for analysis consists of the variables listed in Section II.E. Each of the variables has several levels. For example, there are three levels of relay. The AOV was performed to determine which variables, in addition to those already included in the model, significantly affected the CEP and thus needed to be added to the model. Stepwise Linear Regression is the technique proposed to evaluate the effects of the variables included in the model on the CEP.

2. Analysis of Variance (AOV)

An initial examination of the data in Table 2 suggested a difference in the CEP of each contractor. The CEP for the HAC PLRS test data was substantially lower in five of the eight subsets of data. An AOV was run on all the test data to confirm this difference. A separate AOV was run for both CEP_A and CEP_P using the same independent variables. Also included in this AOV were the variables aspect angle and relay level. These variables were included in the initial model and thus it seemed reasonable to test whether they had a significant effect on the CEP. The number of units LOS, n, although included in the initial model, was not included in this AOV.

The inclusion of n would have caused many cells in the AOV design to be empty. Too many empty cells may have degraded the AOV procedure [9]. The aspect angle was subdivided into four subsets. This subdivision was done with the objective of having equal numbers of data points in each subset. This subdivision helped equalize the number of data points in each cell of the AOV design. Finally, the proportionality of the sample mean to the sample standard deviation was discussed in Section II.E. This proportionality implies that the population mean is proportional to the population standard deviation. If not corrected, this would lead to a violation of the homogeneity of error assumption in the AOV design. Kirk [7] recommends a logarithmic transformation of the dependent variable to remove the proportionality of the population mean to the population standard deviation. This transformation was performed and the resulting model assumed became:

$$\log CEP_{ijkl} = \mu + c_i + a_i + r_k + \varepsilon_{ijkl}$$

where CEP = Circular Error Probable

μ = grand mean

c_i = contractor, i = 1, 2

a_i = aspect angle, i = 1, 2, 3, 4

r_k = relay level, k= 0, 1, 2

ε_{ijkl} = experimental error

The author believed that another variable, not included in this model, could have a significant effect upon the CEP. This variable was the "number of FRU's." It seemed reasonable to assume that the more units with "known" location in the PLRS the lower would be the CEP. By using the "number of FRU's" as an independent variable in the AOV analysis, its significance could be assessed for each contractor. The model assumed was:

$$\log \ CEP_{ijk} = \mu + a_i + F_i + \varepsilon_{ijk}$$

where CEP = Circular Error Probable Accuracy

μ = grand mean

a_i = aspect angle, i = 1, 2, 3

F_i = number of FRU's, i = 1, 2, 3, 4

ε_{ijk} = experimental error

Data was available for which all the variables in the model were fixed while the relay level varied, except for the variable aspect angle. To help remove this aspect angle/relay level confounding, the data for aspect angle was subdivided into three subsets with the number of data points being approximately equal in each of the three subsets.

3. Proposed Model

The initial model used four of the PLRS sources of error discussed in Section II.C. The fifth source of error listed, a unit's speed and altitude, was not included in the model. There was not an obvious form of relationship between the CEP of a unit and a unit's speed and altitude. However,

the author believed that the speed and altitude of a unit could affect the CEP of a unit. The number of FRU's was also believed to affect accuracy. Although there were many ways in which these variables could be included in such a model, for reasons of tractibility these variables were added to the model in the following manner:

$$CEP_i^2 = \frac{k(r_i + 1)}{n_i \sin \alpha^*} (S_i + 1)^{\gamma} (a_i + 1)^{\delta} (F_i)^{\omega}$$

where

CEP_i^2 = CEP^2 of the i^{th} UU

k = range measurement variance

r_i = relay level of i^{th} UU

$\alpha^* = \frac{\alpha}{4}$

n_i = the number of UU's LOS to the i^{th} UU

S_i = the speed of the i^{th} UU

a_i = the altitude of the i^{th} UU

F_i = the number of FRU's in the subtest in which the CEP_i^2 was computed.

The integer one was added to the speed, altitude and relay level terms in the model in order for the model to be valid when any of these variables were equal to zero. The term for which the variable was zero thus would not force CEP^2 to zero, which was not desired.

4. <u>Stepwise Linear Regression</u>

The purpose of using the stepwise linear regression technique was two-fold. First, this technique involves use

of the Multiple Coefficient of Determination, R^2. By using R^2, the relative significance on CEP of the variables in the model could be assessed. Secondly, stepwise linear regression is a well-known procedure for which computer packages are readily available for the analysis. The stepwise linear regression technique requires that a model be linear in the coefficients. The proposed model listed above can be made linear in the coefficients by taking the logarithm of the terms on both sides of the equal sign. The model then takes the form:

$$2LNCEP_i = LNK - LN(\text{Sin } \alpha^*) - LN(n) + \gamma LN(S_i+1)$$
$$+ \delta LN(a_i+1) + LN(R_i+1) + \omega LN(F_i)$$

An estimated prediction equation for this model takes the form:

$$\widehat{2LNCEP}_i = B_0 + B_1 LN(\text{Sin } \overset{*}{\alpha}) + B_2 LN(n_i) + B_3 LN(S_i+1)$$
$$+ B_4 LN(A_i+1) + B_5 LN(R_i+1) + B_6 LN(F_i) + \varepsilon_i$$

The values of the coefficients B_0, B_3, B_4 and B_6 in the model are estimates of model parameters and as such the sign and magnitude of the coefficients are meaningful. The values of the coefficients B_1, B_2 and B_5 do not estimate model parameters but have important meaning. An interpretation of the estimate or meaning of the variable coefficient is presented below.

B_0 : B_0 is an estimate of the logarithm of the range measurement variance plus the variance introduced at each relay level. B_0 should be positive.

B_1 : As the aspect angle increases, the LN Sin α^* also increases. As the LN Sinα^* increases, the CEP should decrease, thus implying that B_1 should be negative.

B_2 : As the number of units LOS increases, the CEP is expected to decrease, thus implying that B_2 should be negative.

B_3 : B_3 is an estimate of γ , the exponent of the variable speed in the model.

B_4 : B_4 is an estimate of δ, the exponent of the variable altitude in the model.

B_5 : The CEP of a unit is expected to increase as the unit moves to higher relay levels, thus implying B_5 should be positive.

B_6 : B_6 is an estimate of ω , the exponent of the variable "number of FRU's" in the model.

III. ANALYSIS AND RESULTS

A. GENERAL

BIOMED 05V and BIOMED 02R were the programs used for the AOV procedure and the stepwise linear regression procedure. In this section the results of the AOV are presented. These results suggested that a different model be used for each contractor's data. The results of the stepwise linear regression are then presented.

B. AOV RESULTS

AOV summary tables for the AOV run with the variables contractor, aspect angle and relay level for the pooled data from both contractors are provided in Tables III and IV. In all cases these variables had significant effects on explaining the variation in both CEP_A and CEP_P. AOV summary tables for the AOV run with the variable aspect angle and the variable "number of FRU's" are provided in Tables V and VI. Data Set 1 from each contractor was used for these analyses. The results of these AOV were quite surprising. The aspect angle had a significant effect in explaining the variation in CEP for both contractors as expected. However, the variable "number of FRU's" was significant only for the GDC PLRS test data. This led to a deletion of the variable "number of FRU's" from the proposed model for the HAC PLRS analysis.

SOURCE	SS	DF	MS	F
Contractor	35.58	1	35.58	107.32*
Angle	32.37	3	10.79	32.55*
Relay Level	44.09	2	22.05	66.50*
Error	203.18	613	.332	

*Significant at the .05 Level

TABLE III. AOV Summary Table Using the CEP_A

SOURCE	SS	DF	MS	F
Contractor	12.23	1	12.23	19.83*
Angle	49.90	3	16.63	26.97*
Relay Level	63.94	2	31.97	51.84*
Error	378.10	613	.617	

*Significant at the .05 Level

TABLE IV. AOV Summary Table Using the CEP_P

SOURCE	SS	DF	MS	F
Aspect Angle	1.93	2	.97	13.13*
Number of FRU's	.073	3	.024	.331
Error	6.54	89	.074	

*Significant at the .05 Level

TABLE V. AOV Summary Table Using the CEP_A Data From The HAC PLRS Test Data

SOURCE	SS	DF	MS	F
Aspect Angle	5.10	2	2.55	39.86*
Number of FRU's	1.26	3	.42	6.54*
Error	5.89	92	.064	

*Significant at the .05 Level

TABLE VI. AOV Summary Table Using the CEP_A Data From The GDC PLRS Test Data

C. REGRESSION RESULTS

1. Multiple Coefficient of Determination

The Multiple Coefficient of Determination, R^2, has numerical value between zero and one. A value close to one indicates that a variable in a linear model "explains" well the deviation in the dependent variable. A value of R^2 close to zero indicates a poor "explanation" of the deviation of the dependent variable. For each regression, an R^2 value is presented. This value is the sum of the contributions to R^2 of all the independent variables included in the regression. This value of R^2 indicates the adequacy of the model in explaining the deviation of the CEP for the particular data set.

2. Results for the HAC PLRS Test Data

Table VII presents the values of the estimated coefficients of the regression model for both CEP_A and CEP_P. Table VIII presents the total value of R^2 for each data set and each variables contribution to the total R^2 value. The significant results of the analysis of this data are listed below:

a. The value of R^2 for the variable relay level accounts for between 30% and 98% of the total R^2 value for the data sets in which relay level was a factor. The value of the estimated coefficient was always positive, indicating that, as expected, as a unit moves to higher relay levels the CEP for that unit increases.

CEP_A

Data Set	B_0	B_1	B_2	B_3	B_4	B_5
1	5.30	-.556	-1.01	.255	.22	2.08
2	-7.72		5.23	2.14	-.86	1.33
3	4.65	-.823	- .52			2.32
4	8.37	-1.12	-3.19			

CEP_P

Data Set	B_0	B_1	B_2	B_3	B_4	B_5
1	2.96	-.24	- .40	.55	.12	2.60
2	-3.58		2.17	1.78	-.39	1.48
3	2.21	.21	.21			2.62
4	4.08	-.99	-1.90			

TABLE VII. Estimated Model Coefficients for the HAC PLRS Test Data

CEP_A

DATA SET

VARIABLE	1	2	3	4
Aspect Angle	.1255		.0028	.3626
Number of UU's LOS	.0154	.0062	.0048	.0165
Relay Level	.2167	.2476	.5472	
Speed	.1528	.5178		
Altitude	.0068	.0642		
Total	.5163	.8358	.5548	.3792

CEP_P

DATA SET

VARIABLE	1	2	3	4
Aspect Angle	.0172		.0086	.4233
Number of UU's LOS	.0018	.0105	.0009	.0081
Relay Level	.2905	.3718	.4973	
Speed	.3235	.4134		
Altitude	.0022	.0007		
Total	.6353	.7964	.5068	.4314

TABLE VIII. Contribution to R^2 of each variable for CEP_A and CEP_P for the HAC PLRS Test Data

b. The value of R^2 for the variable aspect angle accounts for between .5% and 98% of the total R^2 value for the data sets in which the aspect angle was a factor. The value of 98% came from Data Set 4, of which the aspect angle was the variable being examined.

c. The value of R^2 for the variable speed accounts for between 30% and 62% of the total R^2 value for the data sets in which speed was a factor. The values of the estimated coefficients were always positive, indicating that, as expected, as speed increased, the CEP also increased.

d. The value of R^2 for the variables "number of units LOS" and altitude accounted for between .08% and 7.6% of the total R^2 value for the data sets in which these variables were factors. This indicates, somewhat surprisingly, that these variables had very little effect on the CEP.

e. The value of R^2 for CEP_A is higher on Data Sets 2 and 3 but lower on Data Sets 1 and 4 than is the corresponding value of R^2 for CEP_P.

3. Results for the GDC PLRS Test Data

Table IX presents the values of the estimated coefficients of the regression model for both CEP_A and CEP_P. Table X presents the total value of R^2 for each data set and each variables contribution to the total R^2 value. The significant results of the analysis of this data are listed below.

a. The value of R^2 for CEP_A is lower in every data set than is the corresponding value of R^2 for CEP_P.

	Data Set	B_0	B_1	B_2	B_3	B_4	B_5	B_6
	1	4.75	- .85	- .34	.20	.122	1.50	- .111
	2	-1.63		-3.4	2.53	.760	1.39	- .69
CEP_A	3	6.45	- .08				1.30	- .68
	4	4.50	-1.74	-1.40				0.0
	1	2.02	- .94	-1.04	.31	.249	2.67	1.43
	2	-2.61		-2.67	2.65	.441	1.67	- .58
CEP_P	3	5.20	- .10				2.41	-1.54
	4	2.27	-2.26	-2.0				0.0

TABLE IX. Estimated Model Coefficients for the GDC PLRS Test Data

CEP_A

VARIABLE	DATA SET 1	2	3	4
Aspect Angle	.1251		.0017	.5799
Number of UU's LOS	.0013	.0313	.0090	.0019
Relay Level	.1107	.1378	.1934	
Speed	.1161	.3867		
Altitude	.0025	.1021		
Number of FRU's	.0005	.0294		
Total	.3562	.6872	.2041	.5818

CEP_P

VARIABLE	DATA SET 1	2	3	4
Aspect Angle	.1291		.0017	.6630
Number of UU's LOS	.0078	.0248	.0311	.0019
Relay Level	.0854	.3355	.3941	
Speed	.0889	.3609		
Altitude	.0032	.0367		
Number of FRU's	.1540	.0175		
Total	.4683	.7745	.4248	.6649

TABLE X. Contribution to R^2 of each variable for CEP_A and CEP_P for the GDC PLRS Test Data

b. The value of R^2 for the variable relay level accounts for between 18% and 95% of the total R^2 value for the data sets in which relay level was a factor. The value of the estimated coefficient was always positive, indicating that, as expected, as a unit moves to higher relay levels the CEP for that unit increases.

c. The value of R^2 for the variable aspect angle accounts for between .4% and 99% of the total R^2 value for the data sets in which the aspect angle was a factor. The value of 99% came from Data Set 4, of which the aspect angle was the variable being examined.

d. The value of R^2 for the variable speed accounts for between 18% and 53% of the total R^2 value for the data sets in which speed was a factor. The value of the estimated coefficients was always positive, indicating that as speed increased, the CEP also increased.

e. The value of R^2 for the variable "number of FRU's" accounts for between 1.4% and 32.9% of the total R^2 value for the data sets in which the variable "number of FRU's" was a factor. The value 32.9% came from Data Set 1 from the CEP_P data. In all other cases its R^2 value accounted for less than 3% of the total R^2 value.

f. The value of R^2 for the variable "number of units LOS" and the variable altitude accounted for between .3% and 7.3% of the total R^2 value for the data sets in which these variables were factors. This indicates that these variables had very little effect on the CEP.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

There were three variables which had a significant effect upon accuracy in both of the EDM PLRS for both CEP_A and CEP_P. The variables were relay level, aspect angle and speed. This result is not surprising to someone familiar with the PLRS program. However, the author believes that the significant result is found in the variables that were not found significant. The variables altitude, "number of units LOS" and the "number of FRU's" were not found to significantly affect a unit's CEP. This finding certainly could have an effect on user unit employment and placement.

The model form developed has the capability to predict user unit accuracy. An examination of the R^2 values found in Tables VIII and X indicate that in most cases the model adequately "fits" the data. This implies that the model would adequately predict user unit accuracy as a function of the independent variables. However, if the PLRS is redesigned, the parameters of the model would need to be reestimated for it to be an adequate predictor of unit accuracy.

B. RECOMMENDATIONS

There are two main areas that are recommended for future analysis or testing. First, the altitude error of user units was not addressed in this thesis. Research into the optimal employment of user units to minimize the altitude error has

not been undertaken to the author's knowledge but could significantly aid in future PLRS test planning. Secondly, testing has not been done in the sensitivity of user unit accuracy to the position location error of the FRU's. If this system is to be tactically employed where first order survey points are not available, then a complete understanding of this position error is desirable.

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